Risk Premium for Insurance Product Pricing

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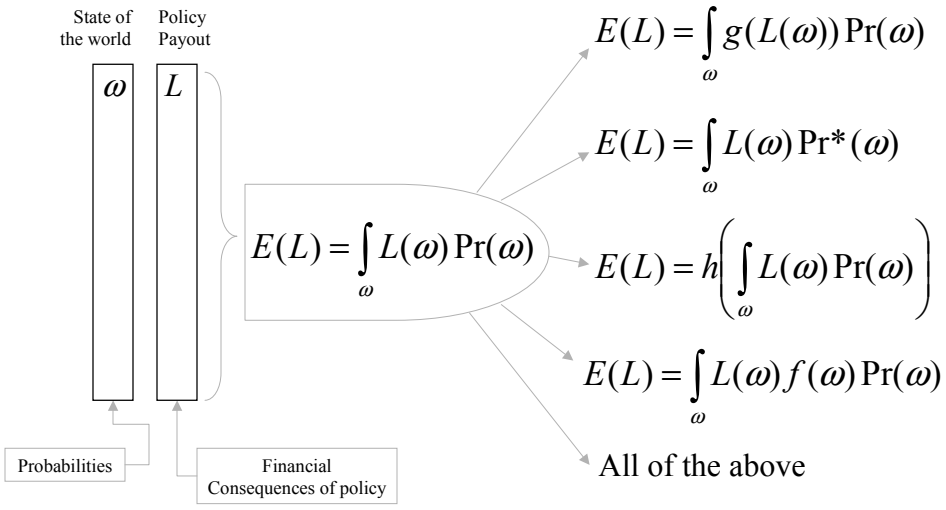
CAS/SOA ERM Symposium
Washington DC, July 2003

Why a Risk Premium?

- Need to make a profit
- Need to be reasonably confident of making a profit
- Risk Premium is an all encompassing term
  - Covers frictional costs
  - Covers pure risk (toss of fair coin)
  - Compensation for bearing risk under uncertainty
- Philosophical distractions should be resisted

Risk Premium: 2000BC-today



Risk Premium

- Standard deviation
- Variance
- Semi-Variance
- Percentile/VaR
- Tail-VaR
- Wang Transform
- Esscher Transform
- Utility-based

- Micro-view of single risk
- SD, Variance,… of what?
- Which measure of risk is appropriate?

Measures of Risk

- Problem: collapse distribution to a number
  - All moments may not be enough to determine distribution!
- No consensus methodology
- Rothschild-Stiglitz offer four possible definitions of when X is "more risky" than Y
  1. $X = Y +$ noise
  2. Every risk averter prefers Y to X (utility)
  3. X has more weight in the tails
  4. $\mathrm{Var}(X) > \mathrm{Var}(Y)$

1, 2, and 3 are equivalent and are different from 4

Parameter Risk: don't delude yourself

- Risk of losses in *your model* is not the same thing as risk of losses!
  - Hayne's Loss Reserving Example (CLRS)
- Leverage, Excess Policies and Jensen's inequality

$$\mathrm{E}(f(X)) \neq f(\mathrm{E}(X))$$

  - Need to compute the mean correctly
  - Risk load should not be used to compensate for miscellaneous actuarial inadequacies

Don't believe a risk load formula that says a new small line is a good thing!

Size: what is a large risk?

- Parameter risk is all that matters…almost
- Process risk matters for large risks
- Large?
  - 100M households in US
  - $1M loss = 1¢ per household
  - $100M loss = $1 per household
  - $1B loss = $10 per household
  - $10B loss = $100 per household

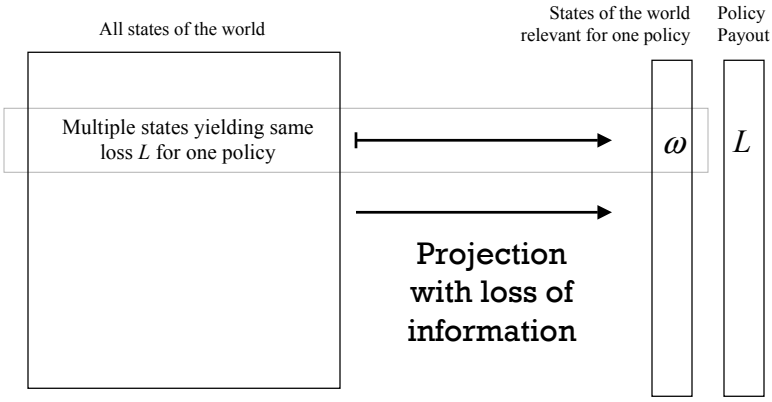
} Large

Size: what is a large risk?

- Heterogeneous distribution of wealth
- Demographics
  - Ultimate risk bearers are individual insureds
  - Population concentrations correlated to risk loads
- Frequency of losses, size of market

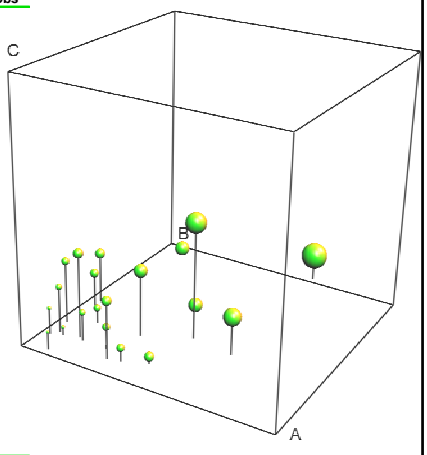
Don't believe a risk load formula that does not account for population demographics

Big Picture: moving beyond individual policy risk



Big Picture: moving beyond individual policy risk

Sim#	A	B	C	Total	Transf Probs
1	77,490	123,643	12,301	213,435	0.160
2	58,089	44,276	54,757	157,122	0.104
3	78,255	41,085	18,167	137,507	0.086
4	8,934	115,909	1,317	126,160	0.074
5	45,939	66,417	2,677	115,033	0.066
6	37,614	34,151	31,340	103,105	0.060
7	5,379	50,342	24,204	79,925	0.054
8	16,600	19,034	40,084	75,717	0.050
9	36,492	10,658	27,340	74,490	0.046
10	53,382	16,521	3,671	73,574	0.042
11	6,911	43,635	17,632	68,179	0.039
12	42,304	12,079	6,515	60,898	0.036
13	4,114	26,544	29,910	60,568	0.033
14	31,730	16,976	10,725	59,431	0.030
15	15,796	33,017	7,587	56,401	0.027
16	19,180	17,466	13,622	50,268	0.025
17	6,756	18,021	22,012	46,789	0.022
18	3,967	14,584	12,489	31,040	0.019
19	9,401	16,660	3,956	30,017	0.016
20	11,352	3,810	8,277	23,439	0.011
Mean	28,484	36,241	17,429	82,155	
Loaded	40,416	53,667	19,733	113,815	
Load	42%	48%	13%	39%	
SD	23,584	31,804	13,564	46,310	
CV	82.8%	87.8%	77.8%	56.4%	



Big Picture: moving beyond individual policy risk

- Finance: state price densities
  - $f(\omega)$ from slide 3
- Rodney Kreps, co-measures

  $$\mathrm{E}\left(g(X_i)\middle|\text{ condition on }\sum X_i\right)$$
- P/C: Catastrophe (re-)insurance
  - Cat models explicitly quantify correlation
- Life: Hedging interest rate and investment risk

Three Points to Remember

- Parameter Risk
- Size
- Think Big-Picture